Feedback Control of Tethered Satellites Using Lyapunov Stability Theory

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This paper treats the three-dimensional aspects of tethered satellite deployment and retrieval. Feedback control laws with guaranteed closed-loop stability are obtained using the second method of Lyapunov. Tether mass and aerodynamic effects are not included in the design of the control laws. First, a coordinate transformation is presented that partially uncouples the in-plane and out-of-plane dynamics. A combination of tension control as well as out-of-plane thrusting is shown to be adequate for fast retrieval. Next, a unified control design method based on an integral of motion (for the coupled system) is presented. It is shown that the controller designed by the latter method is superior to that of the former, primarily from the out-of-plane thrust usage point of view. A detailed analysis of stability of the closed-loop system is presented and the existence of limit cycles is ruled out if out-of-plane thrusting is used in conjunction with tension control. Finally, a tether rate control law is also developed using the integral of motion mentioned previously. The control laws developed in the paper can also be used for stationkeeping.

Introduction

CURRENTLY, many space missions involving tethers are being planned. One of these is the Tether Dynamics Explorer (TDE-1) mission scheduled for July 1991. TDE-1 will deploy a 22.7-kg rectangular subsatellite attached to the spent second stage of a Delta-II launch vehicle using a 20-km tether.[1] The main aim of this mission is to validate the tether deployment performance of various control laws proposed to date. The TSS-1 (Tethered Satellite System) mission is also being planned for 1991 for conducting electrodynamic research. Space tethers can also be utilized for study of the lower atmosphere, micro and variable g experiments, space construction, and gravity gradient stabilization.

One of the primary issues in tether utilization is fast deployment/retrieval of attached payloads. Rupp[2] provided the impetus for the study of tether dynamics and control law development in 1975 and since then a vast body of literature has come to exist. An excellent survey of the literature has been conducted by Misra and Modi.[3] Rupp's control law was originally designed for in-plane deployment and utilized a feed-forward tether length command as well as linear feedback of length and length rate. Later studies have proposed control laws for deployment and retrieval involving additional linear/nonlinear feedback of in-plane pitch angle and its rate, the out-of-plane roll angle and its rate, and tether extensional as well as flexural modes.[4-6] Liangdong and Bainum[6] have also investigated the effect of tether mass and flexibility and the gains of the tension control law on the (in-plane) stability of stationkeeping. In Ref. 6, they show that the stability conditions involving the length and rate gains for a flexible tether are qualitatively similar to those for a rigid tether.

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It has been concluded that deployment can be controlled with relative ease but retrieval is more difficult to control as large amplitude in-plane as well as out-of-plane tether librations are excited and sufficient tension cannot be maintained during terminal retrieval phases. Thruster augmentation has been suggested by Banerjee and Kane[7] to overcome these difficulties. Retrieval utilizing tether-normal thrusting and based on shuttle orbiter maneuvering and sliding mode control has been proposed by Pines et al.[8] A mechanism by which the subsatellite crawls on the tether has also been proposed by Kane.[9] A comparison of tension controlled retrieval and retrieval using the crawler mechanism has been conducted by Glickman and Rybak,[10] and it is shown that the latter technique has several advantages including low levels of libration amplitudes and fast terminal retrieval rates. The disadvantage of this method is that, if the tether is not retrieved, it may have to be jettisoned. This will add to the already serious space debris problem.

Even if characteristics such as tether flexibility and atmospheric effects are neglected, the equations of motion for a tethered satellite are highly nonlinear, nonautonomous (if the tether retrieval/deployment rate is specified as a function of time), and coupled. Hence, the synthesis of a suitable nonlinear control law is difficult. A Lyapunov (mission function) approach has been used for tether deployment and retrieval by Fujii and Ishijima.[11] Tether mass and flexibility as well as aerodynamic effects are neglected in this study. The proposed nonlinear tension control law has been designed for controlling deployment and retrieval in the orbital plane. It is based on feedback of tether length, length rate, pitch angle, and pitch rate. A feed-forward length command is not needed. In an alternate treatment of the same problem, Vadali[12] concludes that, under similar assumptions, a linear feedback of tether length and its rate is sufficient to guarantee asymptotic stability of the closed-loop system to the desired equilibrium point. Fast retrieval is possible if the pitch angle is not actively controlled to be near its equilibrium value but allowed to deviate sufficiently either in the mid or terminal phases of retrieval.

This paper treats the three-dimensional aspects of tethered satellite deployment and retrieval. Feedback control laws with guaranteed closed-loop stability are obtained using the second method of Lyapunov. Tether mass and aerodynamic effects are not included in the design of the control laws. First, a coordinate transformation is presented that partially uncouples the in-plane and out-of-plane dynamics. A combination of tension control as well as out-of-plane thrusting is shown to be adequate for a speedy retrieval. Next, a unified control design method based on an integral of motion (for the coupled system) is presented. It is shown that the controller designed by the latter method is superior to that of the former primarily from the out-of-plane thrust usage point of view. A detailed analysis of stability of the closed-loop system is presented and existence of limit cycles is ruled out if out-of-plane thrusting is used in conjunction with tension control. Finally, a tether rate control law is also developed using the integral of motion mentioned earlier. The control laws developed in the paper can also be used for stationkeeping.

Lyapunov Approach

In many instances, one can consider control and stabilization to be equivalent. Global asymptotic stability can be ascertained by using Lyapunov's second method.[13] Choosing the right Lyapunov function is a difficult task but one can sometimes find suitable positive definite energy or Hamiltonian functions based on the principles of analytical dynamics.[14] The beauty of the method is that it is not based on linearization.

The Lyapunov approach is briefly outlined next. Let the dynamic system be described by the system of nonlinear, ordinary differential equations

$$\dot{x} = f(x,u,t) \tag{1}$$

where x is the state vector, u the control vector, and t denotes time. The desired final state is assumed to be the origin of the state space. This framework is still valid if a nontrivial equilibrium point is desired, for the origin can be placed there by a suitable coordinate transformation. Let V be a positive definite function. The time derivative of V can be written as

$$\dot{V} = \left[\frac{\partial V}{\partial x}\right]^T f + \frac{\partial V}{\partial t} \tag{2}$$

If this preceding identity can be solved for u as a function of x and t, such that $\dot{V}$ is globally negative definite, a feedback control law is obtained that renders the closed-loop system globally, asymptotically stable. Of course, the nature of the control law depends on the type of V chosen. If $\dot{V}$ can only be made negative semidefinite, it must be verified that this $\dot{V}$ remains zero only at the desired final state.

The earliest work utilizing this method has been presented by Kalman and Bertram.[15] Later applications of this method for controlling spacecraft attitude maneuvers can be found in Refs. 16-22. A different approach is followed by Lee and Grantham,[23] in which the directional derivative of V in the direction of f is minimized using the method of steepest descent. This procedure has benefits of optimality and the ability to enforce actuator saturation constraints, but it typically involves on-line computation of roots of a polynomial/transcendental equation.

Equations of Motion

In this study, it is assumed that the tether is rigid, of negligible mass, and remains straight. Tether flexibility and elasticity, as well as the subsatellite motion and aerodynamic effects are neglected. The equations of motion of the tether and the attached satellite[4] are

$$\ddot{l} - l[\dot{\phi}^2 + \cos^2\phi(\dot{\theta} + \Omega)^2 - \Omega^2 + 3\Omega^2 \cos^2\phi \cos^2\theta] = -T/m \tag{3a}$$

$$\ddot{\theta} + 2[(\dot{l}/l) - \dot{\phi}\tan\phi](\dot{\theta} + \Omega) + 3\Omega^2 \cos\theta \sin\theta = 0 \tag{3b}$$

$$\ddot{\phi} + 2(\dot{l}/l)\dot{\phi} + \cos\phi \sin\phi[(\dot{\theta} + \Omega)^2 + 3\Omega^2 \cos^2\theta] = F/(ml) \tag{3c}$$

where l indicates the instantaneous tether length, θ the pitch angle (in plane), ϕ the roll angle (out-of-plane), Ω the orbital rate, T the tension, F the out-of-plane thrust, and m the mass of the subsatellite. These equations can be nondimensionalized by defining the following nondimensional variables:

$$\tau = \Omega t, \qquad \lambda = \frac{l}{L}, \qquad \hat{T} = \frac{T}{[m\Omega^2 L]}, \qquad \hat{F} = \frac{F}{[m\Omega^2 L]}$$

where L is the reference tether length. The nondimensional equations are

$$\lambda'' - \lambda[\phi'^2 + \cos^2\phi(1+\theta')^2 - 1 + 3\cos^2\phi\cos^2\theta] = -\hat{T} \tag{4a}$$

$$\theta'' + 2[(\lambda'/\lambda) - \phi'\tan\phi](1+\theta') + 3\cos\theta\sin\theta = 0 \tag{4b}$$

$$\phi'' + 2(\lambda'/\lambda)\phi' + \cos\phi\sin\phi[(1+\theta')^2 + 3\cos^2\theta] = \hat{F}/\lambda \tag{4c}$$

where the prime indicates the derivative with respect to nondimensional time. $\hat{T}$ and $\hat{F}$ are treated as the control variables, and methods for designing control laws are discussed in the following sections. A tether rate control law is also developed by treating λ' as the control variable instead of $\hat{T}$.

Integral of Motion

Before proceeding with a choice of a Lyapunov function, the existence of integrals of motion should be examined. Such integrals for the linearized in-plane motion have been derived by Rajan and Anderson[24] using Noether's theorem. Integrals of motion of the nonlinear system of equations are of interest here. Since the system is autonomous, the Hamiltonian is conserved. The Hamiltonian has been used as a Lyapunov function in many stability analyses.[25] However, positive definiteness of the Hamiltonian has to be ascertained before it can be used as a Lyapunov function.[14] The nondimensional Hamiltonian can be written as

$$H = \tfrac{1}{2}[\lambda'^2 + \lambda^2(\theta'^2\cos^2\phi + 3\sin^2\theta\cos^2\phi + \phi'^2 + 4\sin^2\phi - 3)] \tag{5}$$

It is clear that, if the tether length is fixed and $\hat{F}$ is zero, then the following positive definite function

$$V_1 = \tfrac{1}{2}(\theta'^2\cos^2\phi + 3\sin^2\theta\cos^2\phi + \phi'^2 + 4\sin^2\phi) \tag{6}$$

is conserved. It can be shown that if the tether length is not constant,

$$V_1' = -2(\lambda'/\lambda)[\theta'(1+\theta')\cos^2\phi + \phi'^2] \tag{7}$$

This is an important result, since it can be used to determine maximum libration amplitudes analytically and, even more, it can be said that if λ'/λ is extremely small, the tether librations will closely resemble limit cycles. Figure 1 shows a roll angle vs pitch angle plot for a constant tether length ($\lambda = 1.0$) and initial conditions $\theta = \phi = 5$ deg and $\theta' = \phi' = 0$. The function

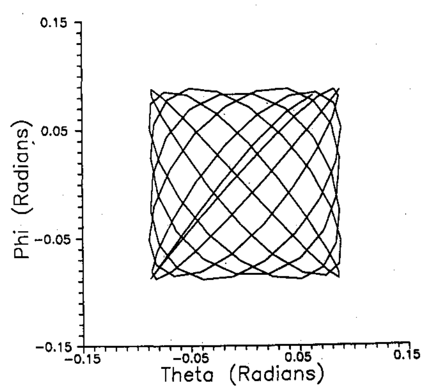

Fig. 1 Pitch-roll librations (constant length tether).

V_1 given earlier will be used extensively in defining candidate Lyapunov functions in this paper.

A tether rate control law can be developed easily to damp the pitch and roll librations. The tether rate may be oscillatory depending on the initial conditions. A stationkeeping strategy based on modulating the tether length has been proposed by Davis and Banerjee.[26] It is also seen from Eq. (7) that for the function V_1 to decrease, λ' has to be positive for $\theta' \geq 0$. On the other hand, for very small values of ϕ and ϕ' and small negative values of θ',λ' has to be negative for V_1 to decrease. This suggests that, if θ is allowed to increase initially in a positive sense and ϕ and ϕ' are small, then a unidirectional retrieval is possible without tether oscillations. This further implies that a unidirectional retrieval is possible if out-of-plane thrust is utilized to control roll and roll rate.

In-Plane Control Law

A control law for in-plane deployment and retrieval is first reviewed for completeness. If ϕ and $\dot{\phi}$ are assumed to be zero at the initial time and $\hat{F}$ is zero, only the in-plane equations are needed to describe the motion of the tether. This is also true if the out-of-plane motion is actively controlled. The in-plane equations of the tether are

$$\lambda'' - \lambda[(1+\theta')^2 - 1 + 3\cos^2\theta] = -\hat{T} \tag{8a}$$

$$\theta'' + 2(\lambda'/\lambda)(1+\theta') + 3\cos\theta\sin\theta = 0 \tag{8b}$$

If downward deployment and upward retrieval are considered, the desired final boundary conditions on the pitch angle and its rate are $\theta = \theta' = 0$. A simple control law applicable in this situation is based on the following Lyapunov function[12]:

$$V = \tfrac{1}{2}[\lambda'^2 + K_1(\lambda - \lambda_f)^2 + (K_2 + \lambda_2)(\theta'^2 + 3\sin^2\theta)] \tag{9}$$

where $\lambda_f > 0$ is the desired final value of λ. K_1 is a positive constant and K_2 can either be positive or zero. The nature of the Lyapunov function is such that the undesirable conditions $\theta = \pi$ and $\theta' = 0$ can also be reached. This Lyapunov function has some similarity to that used by Fujii and Ishijima.[11] The nondimensional tension control law is

$$\hat{T} = 3\lambda + K_1(\lambda - \lambda_f) - 2K_2\theta'(1+\theta')/\lambda + K_3\lambda' \tag{10}$$

It can be verified that this feedback control law locally asymptotically stabilizes the closed-loop system in the neighborhood of the desired final conditions. Care must be exercised in selecting the gains as the tension must remain positive and the undesirable equilibrium point must be avoided. A particularly interesting special case is obtained if K_2 is set to zero. The control law then feeds back the instantaneous tether length and its rate. This is a continuous equivalent of Rupp's control law without the discrete feed-forward commands. It is observed that the stability conditions derived by Liangdong and Bainum[6] for Rupp's control law ($K_1 > 0$, $K_2 = 0$, and $K_3 > 0$) are clearly satisfied by Eq. (10). A nonzero K_2 is effective in suppressing the pitch deviations but the tether length response slows down considerably. For nondimensional tether lengths below 0.01, better performance is obtained with K_2 set to zero.

Presence of pitch as well as roll motions is considered next. Out-of-plane thrust is utilized to keep roll motion bounded. Tension, rate, and out-of-plane thrust control laws are developed in the following sections using the Lyapunov approach.

Tension Control Law Design Based on Decoupled Equations of Motion

The first method is based on a coordinate transformation that nearly uncouples the in-plane and out-of-plane motions. If we define $z = \lambda\cos\phi$ and $y = \lambda\sin\phi$, the differential equations [Eqs. (4)] transform to

$$z'' - z[(1+\theta')^2 - 1 + 3\cos^2\theta] = -(\hat{T}/\lambda)z - (\hat{F}/\lambda)y \tag{11a}$$

$$\theta'' + 2(z'/z)(1+\theta') + 3\cos\theta\sin\theta = 0 \tag{11b}$$

$$y'' + y = -(\hat{T}/\lambda)y + (\hat{F}/\lambda)z \tag{11c}$$

It will be useful to note the following relationships:

$$yy' + zz' = \lambda\lambda' \tag{12a}$$

$$zy' - yz' = \lambda^2\phi' \tag{12b}$$

The nonhomogeneous part of each of these equations can be treated as a generalized force. It is interesting to note that Eqs. (11a) and (11b) are similar to Eqs. (8). Equation (11c) has the form of a linear oscillator with a forcing function. Thus, the in-plane motion can be controlled by using a modified version of the tension control law given by Eq. (10), and it is a simple matter to control the out-of-plane motion using derivative feedback for the generalized out-of-plane force. Hence, the following control laws are chosen:

$$\begin{bmatrix} z & y \\ y & -z \end{bmatrix} \begin{Bmatrix} \hat{T}/\lambda \\ \hat{F}/\lambda \end{Bmatrix} = \begin{Bmatrix} 3z + K_1(z - z_f) - 2K_2\theta'(1+\theta')/z + K_3 z' \\ K_3 y' \end{Bmatrix} \tag{13}$$

Equation (13) can always be solved for $\hat{T}$ and $\hat{F}$ because the determinant of the matrix to be inverted is $-\lambda^2$. The control laws can be written in terms of the original variables using Eqs. (12), as

$$\hat{T} = (3 + K_1)\lambda\cos^2\phi - K_1\lambda_f\cos\phi - 2K_2\theta'(1+\theta')/\lambda + K_3\lambda' \tag{14}$$

$$\hat{F} = (3 + K_1)\lambda\cos\phi\sin\phi - K_1\lambda_f\sin\phi - 2K_2\theta'(1+\theta')/\lambda\tan\phi + K_3\lambda\phi' \tag{15}$$

In what follows, retrieval of a tethered satellite is considered. The primary body is the Space Shuttle assumed to be in a circular orbit at an altitude of 220 km, with an orbital rate of 0.07068 rad/min. The orbital period is nearly 1.48 h. The tether is assumed to be 20 km long. The initial conditions for the motion of the tether are $\lambda = 1.0$, $\lambda' = 0$, $\theta = \phi = 5$ deg, and $\theta' = \phi' = 0$, and the final conditions are $\lambda = 0.01$ and $\lambda' = \theta = \theta' = \phi = \phi' = 0$. Note that, if an exponential feed-forward command is used, the initial velocity has to be finite.

Figure 2 shows the variations of tether length, pitch angle, and tension, and Fig. 3 shows the roll angle and out-of-plane thrust variations during retrieval for gain settings $K_1 = 1.0$, $K_2 = 0$, and $K_3 = 0.3$. The results indicate that the tether length and pitch angle reach their respective final values in nearly two orbits but the roll angle response is slower. The reason for this uncoupled behavior can be explained as a result of the decoupled control design. The pitch angle undergoes a sharp change near the end of the retrieval. The initial nondimensional tension is 4, the equilibrium initial value is 3. In dimensional form, assuming a satellite mass of 22.7 kg, the initial tension is 2.52 N. This value will be higher if tether mass is included. It should be noted that this example depicts a fast retrieval. The initial value of tension can be decreased further for a slower retrieval. Thrust usage has been quantified by evaluating the index

$$\int \left| \hat{F} \right| \mathrm{d}\tau$$

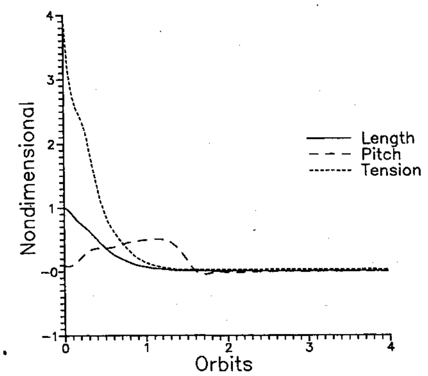

Fig. 2 Retrieval using controller 1, out-of-plane thrust used: $K_1 = 1.0$, $K_2 = 0$, and $K_3 = 3.0$.

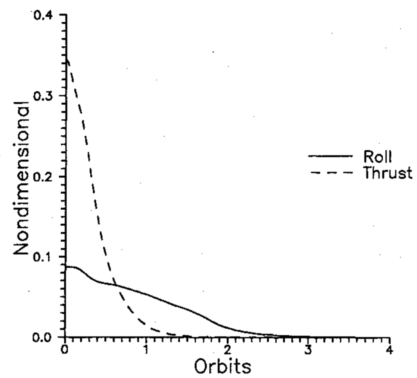

Fig. 3 Retrieval using controller 2, out-of-plane thrust used: $K_1 = 1.0$, $K_2 = 0$, and $K_3 = 3.0$.

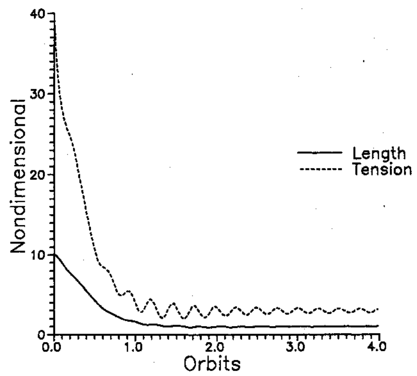

Fig. 4 Retrieval using controller 2, no out-of-plane thrust: $K_1 = 1.0$, $K_2 = 0$, and $K_3 = 3.0$.

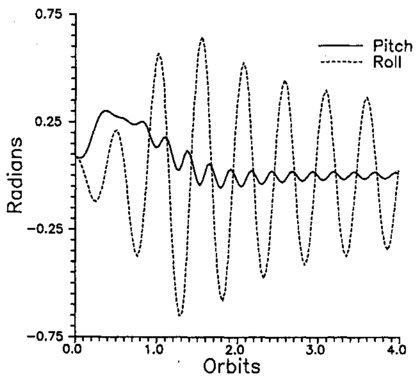

Fig. 5 Pitch and roll angles using controller 2, no out-of-plane thrust: $K_1 = 1.0$, $K_2 = 0$, and $K_3 = 3.0$.

over two orbits. This nondimensional index is 0.9, which amounts to an average thrust impulse of 481 N-s.

Tension Control Law Design Based on Coupled Equations of Motion

The second method is based on the following candidate Lyapunov function:

$$V = \tfrac{1}{2}[\lambda'^2 + K_1(\lambda - \lambda_f)^2 + (K_2 + \lambda^2)(\theta'^2 \cos^2\phi + 3\sin^2\theta\cos^2\phi + \phi'^2 + 4\sin^2\phi)] \tag{16}$$

A significant part of this function is the integral of motion obtained previously. Note that this choice of V automatically admits the possibility of existence of multiple equilibria. They are given by

$$\lambda = \lambda_f, \quad \lambda' = 0, \quad \theta = \theta' = \phi = \phi' = 0$$

$$\lambda = \lambda_f, \quad \lambda' = 0, \quad \theta = \theta' = 0, \quad \phi = \pi, \quad \phi' = 0$$

$$\lambda = \lambda_f, \quad \lambda' = 0, \quad \theta = \pi, \quad \theta' = 0, \quad \phi = \phi' = 0$$

The last two equilibria are one and the same and also undesirable for downward deployment/ upward retrieval. Besides these, other equilibria might exist. This possibility will be investigated subsequently.

The time derivative of V is given by

$$V' = \lambda'\{3\lambda - \hat{T} + K_1(\lambda - \lambda_f) - 2K_2[\theta'(1+\theta')\cos^2\phi + \phi'^2]/\lambda\} + \phi'(K_2 + \lambda^2)\hat{F}/\lambda \tag{17}$$

If we assume that out-of-plane thrust is not utilized, the tension control law can be selected as

$$\hat{T} = 3\lambda + K_1(\lambda - \lambda_f) - 2K_2[\theta'(1+\theta')\cos^2\phi + \phi'^2]/\lambda + K_3\lambda' \tag{18}$$

so that

$$V' = -K_3\lambda'^2 \tag{19}$$

Simulations using Eq. (18) reveal that there is a significant interplay between the tether length and swing motion. If K_2 is set to zero, the roll libration amplitudes become alarmingly high. This can be seen in Figs. 4 and 5. The gains used are $K_1 = 1.0$, $K_2 = 0$, and $K_3 = 3$. The initial and final nondimensional tether lengths are, respectively, $\lambda(0) = 10$ and $\lambda_f = 1$. Other initial and final conditions are same as before. Figure 4 shows the nondimensional length and tension and Fig. 5 shows the pitch and roll angles. The effect of a positive K_2 on the system response is shown in Figs. 6 and 7 for $K_2 = 0.5$. It is observed that the tether retrieval rate is oscillatory near the terminal phase and convergence to the desired equilibrium point is extremely slow. However, the maximum roll libration amplitude is much lower. It should also be noted that, by about one orbital period, the subsatellite is sufficiently close to permit an open-loop retrieval using a boom. This example is of significance as thruster firings in the vicinity of the parent vehicle may not be permissible due to operational safety requirements.

If out-of-plane thrust is utilized, the tension control law need not be changed. Simple rate feedback thrust control is sufficient to enhance the stability of the closed-loop system significantly. The out-of-plane control law is selected to be

$$\hat{F} = -K_4\lambda\phi' \tag{20}$$

Roll angle feedback can also be included in the out-of-plane thrust control law by adding a quadratic term in ϕ to the

Lyapunov function. The usage of the tension as well as the out-of-plane control law leads to

$$V' = -K_3\lambda'^2 - (K_2 + \lambda^2)K_4\phi'^2 \tag{21}$$

It is easy to verify the stability of the closed-loop system. If λ' and ϕ' are both zero, the closed-loop system is given by the following equations:

$$-\lambda[\cos^2\phi(1+\theta')^2 - 1 + 3\cos^2\phi\cos^2\theta] = -3\lambda - K_1(\lambda - \lambda_f)$$
$$+ 2K_2[\theta'(1+\theta')\cos^2\phi]/\lambda \tag{22a}$$

$$\theta''\cos\phi + 3\cos\phi\cos\theta\sin\theta = 0 \tag{22b}$$

$$\cos\phi\sin\phi[(1+\theta')^2 + 3\cos^2\theta)] = 0 \tag{22c}$$

It can be shown that a local equilibrium point is $\lambda = \lambda_f$ and $\lambda' = \theta = \theta' = \phi - \phi' = 0$. Other undesirable equilibria do exist but can be avoided by properly selecting the control gains.

Simulation results using this control law with gains $K_1 = 1.0$, $K_2 = 0$, $K_3 = 3.0$, and $K_4 = 2.0$ are shown in Figs. 8 and 9. The initial and final conditions are the same as those used in simulations with the decoupled control law. It is observed that the retrieval process is unidirectional and quite similar to that obtained using the decoupled control law. However, the required out-of-plane thrust is much less and the roll response is faster. The nondimensional thrust impulse index, defined earlier, evaluated over two orbits is 0.24, which is 128.4 N-s. If K_4 is selected as 3 in the previous example, the thrust impulse index amounts to 0.258.

The desired nondimensional final length of the tether in the previous example is 0.01. The effect of a smaller desired final length (0.001) on the performance of the control laws is investigated next. The control parameters are $K_1 = 1.0$, $K_2 = 0$, $K_3 = 3.0$, and $K_4 = 2.0$. The initial conditions are the same as

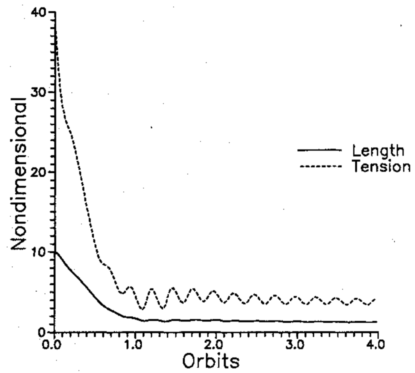

Fig. 6 Retrieval using controller 2, no out-of-plane thrust: $K_1 = 1.0$, $K_2 = 0.5$, and $K_3 = 3.0$.

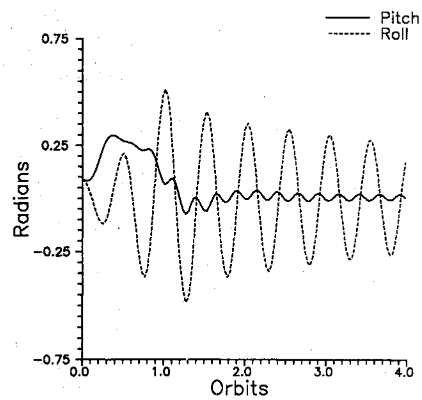

Fig. 7 Pitch and roll angles using controller 2, no out-of-plane thrust: $K_1 = 1.0$, $K_2 = 0.5$, and $K_3 = 3.0$.

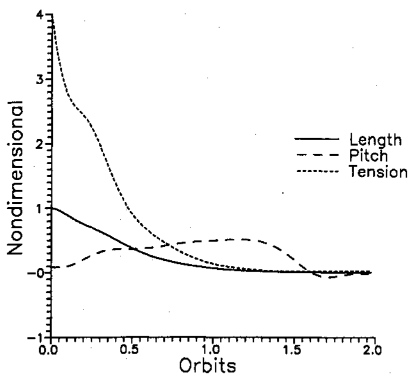

Fig. 8 Retrieval using controller 2, out-of-plane thrust used: $K_1 = 1.0$, $K_2 = 0$, $K_3 = 3.0$, and $K_4 = 2.0$.

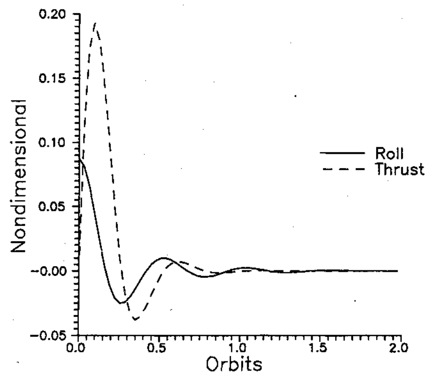

Fig. 9 Thrust and roll angle using controller 2: $K_1 = 1.0$, $K_2 = 0$, $K_3 = 3.0$, and $K_4 = 2.0$.

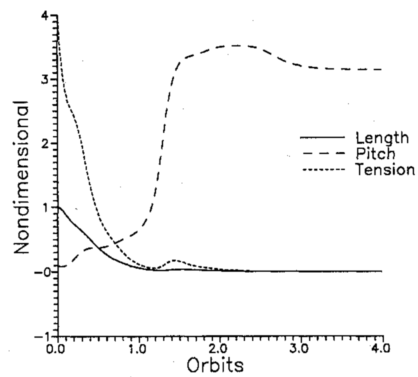

Fig. 10 Undesirable retrieval using controller 2, out-of-plane thrust used: $K_1 = 1.0$, $K_2 = 0$, $K_3 = 3.0$, and $K_4 = 2.0$ ($\lambda_f = 0.001$).

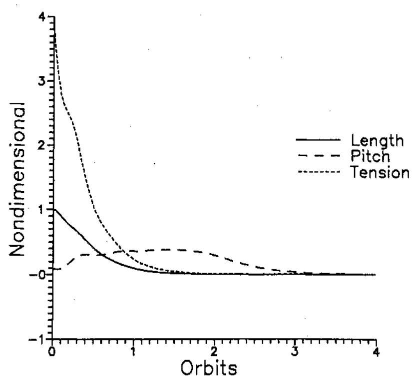

Fig. 11 Desirable retrieval using controller 2, out-of-plane thrust used: $K_1 = 0.9$, $K_2 = 0$, $K_3 = 3.0$, and $K_4 = 2.0$ ($\lambda_f = 0.001$).

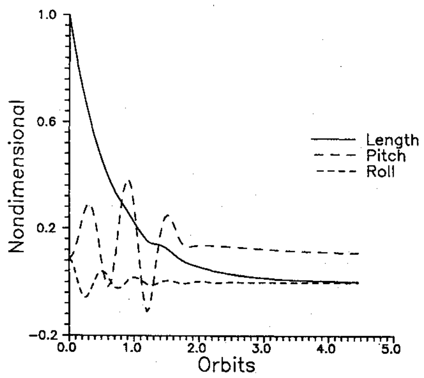

Fig. 12 Retrieval using the rate control law, out-of-phase thrust used: $K_1 = 0.25$, $K_2 = 0.01$, and $K_3 = 1.0$.

before. The tether length, pitch angle, and tension are shown in Fig. 10. The pitch angle is too large and an undesirable equilibrium point is reached. A slight change in the gain K_1 from 1.0 to 0.9 produces a nice retrieval, as shown in Fig. 11. The maximum value of the pitch angle is 21.9 deg in Fig. 11 as compared to 29.3 deg in Fig. 8. An increase in the gain K_2 also serves the same purpose as decreasing K_1.

Tether Rate Control Law

A tether rate control law can be developed easily using the integral of motion for a tether of constant length, given by Eq. (6). Out-of-plane thrust is utilized for this application also. Based on the previous developments, the following candidate Lyapunov function is chosen:

$$V = \tfrac{1}{2}[K_1(\lambda - \lambda_f)^2 + K_2 V_1] \tag{23}$$

The rate control law and the out-of-plane thrust law can be obtained following the usual process as

$$\lambda' = -K_1(\lambda - \lambda_f) + K_2[\theta'(1 + \theta')\cos^2\phi + \phi'^2]/\lambda \tag{24}$$

$$\hat{F} = -K_3 \lambda \phi' \tag{25}$$

If K_2 is very small, the retrieval process is nearly exponential. This leads to a nearly constant (slow decay) pitch angle during the terminal phases of retrieval. Figure 12 shows the tether length and pitch and roll angles for $K_1 = 0.25$, $K_2 = 0.01$, and $K_3 = 1.0$. The initial conditions on the tether motion are the same as before and $\lambda_f = 0.01$. The value of K_1 dictates the initial tether retrieval rate for this example as the initial pitch and roll rates are zero. The choice of K_1 results in an initial dimensional retrieval rate of 30 m/s, which is moderate. The pitch angle behavior for this example is oscillatory, unlike that for the previous examples. The slow decay of the pitch angle is also noted.

Conclusions

Lyapunov feedback control design methods have been presented for deployment and retrieval of tethered satellites. The first method is based on partial decoupling of the equations of motion and utilization of a two-dimensional control law developed previously using Lyapunov stability theory. The second method uses a Lyapunov function based on a first integral of motion of the original set of differential equations. Controllers designed by both methods work very well but the second controller has the advantage of using less out-of-plane thrust. These control laws are quite simple and utilize tether tension control as well as out-of-plane thrusting. Lyapunov stability analysis is used to rule out the possibility of limit cycles. It is recommended that the control gains be chosen such that the pitch angle does not exceed ± 30 deg. This will ensure that undesirable equilibrium points are not reached. The gains in the tension control law should be adjusted according to the desired final tether length. A rate control law derived using the integral of motion is also presented. Finally, it is apparent that controlling the roll librations is essential for retrieval.

Further validation of the effectiveness of these control laws in the presence of tether flexibility and elasticity, aerodynamic effects, and motion of the subsatellite is necessary.

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